

v) Yes, it is a set as it can be expressed as

$$\{1, 2, 3, 4, 5, 6, \dots, 99\}$$

vi) Yes, it is a set.

vii) Yes, it is a set as it can be expressed as

$$\{\dots, -8, -6, -4, -2, 0, 2, 4, 6, 8, \dots\}$$

viii) Yes, it is a set

ix) No, it is not a set as it not well-defined.

2. Let $A = \{1, 2, 3, 4, 5, 6\}$. Insert the appropriate symbol \in or \notin in the blank spaces:

i) $5 \in A$

ii) $8 \notin A$

iii) $0 \notin A$

iv) $4 \in A$

v) $2 \in A$

vi) $10 \notin A$

3. Write the following sets in roster form:

i) $A = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

ii) $B = \{1, 2, 3, 4, 5\}$

iii) $C = \{17, 26, 35, 44, 53, 62, 71, 80\}$

$$\text{iv)} \quad D = \{2, 3, 5\}$$

$$\text{v)} \quad E = \{T, R, I, G, O, N, M, E, Y\}$$

$$\text{vi)} \quad F = \{B, E, T, R\}$$

4. Write the following sets in the set-builder form:

$$\text{i)} \quad \{x : x = 3n, n \in \mathbb{N}, 1 \leq n \leq 4\}$$

$$\text{ii)} \quad \{x : x = 2^n, n \in \mathbb{N}, 1 \leq n \leq 5\}$$

$$\text{iii)} \quad \{x : x = 5^n, n \in \mathbb{N}, 1 \leq n \leq 4\}$$

$$\text{iv)} \quad \{x : x \text{ is an even natural no.}\}$$

$$\text{v)} \quad \{x : x = n^2, n \in \mathbb{N}, 1 \leq n \leq 10\}$$

5. List all the elements of the following sets:

$$\text{i)} \quad A = \{1, 3, 5, 7, \dots\}$$

$$\text{ii)} \quad B = \{0, 1, 2, 3, 4\}$$

$$\text{iii)} \quad C = \{-2, -1, 0, 1, 2\}$$

$$\text{iv)} \quad D = \{L, O, Y, A\}$$

$$\text{v)} \quad E = \{\text{February, April, June, September, November}\}$$

$$vi) F = \{j, k, q, f, d, c, b\}$$

Q. Match the roaster form with its set-builder form.

- i) c
- ii) a
- iii) d
- iv) b
- v)

EQUAL SETS

$$A = \{a, b, c, d\}$$

$$B = \{b, c, d, a\}$$

EMPTY SET / NULL SET / VOID SET

⇒ does not have any elements

⇒ represented as ϕ (phi) / $\{\}$

* Empty Set is always finite

SUBSET

E.g. $A = \{1, 2, 3, 4\}$
 $B = \{1, 2, 3, 4\}$

$$A \subset B \text{ \& } B \subset A$$

$$\therefore A = B$$

& converse is also true.

⇒ 'c' → contained

E.g. $A = \{1, 2, 3, 4, 5, 6, 7\}$
 $B = \{1, 2, 3, 4\}$

$$\therefore B \subset A$$

* Every Set is a Subset of Itself

→ E.g. $A = \{1, 2, 3, 4\}$ $\boxed{\therefore A \subset A}$

* ϕ is a subset of every set

E.g. $A = \{1, 2, 3, 4, 5, \dots, 10\}$

$B = \{5, 6, 7, 8, 9, 10\}$

$\boxed{\therefore B \subset A}$

subset

superset

SINGLETON SET

→ a set which has only one element is known as a Singleton set.

INTERVALS

$[1, 2]$ → square = closed interval

$(1, 2)$ → open interval

$[1, 2] \Rightarrow \leftarrow \left[\begin{array}{c} | \\ 1 \\ | \end{array} \quad \begin{array}{c} | \\ 2 \\ | \end{array} \right] \rightarrow$

$(1, 2) \Rightarrow \leftarrow \left(\begin{array}{c} | \\ 1 \\ | \end{array} \quad \begin{array}{c} | \\ 2 \\ | \end{array} \right) \rightarrow$

Exercise.1.2

1. Which of the following are examples of null set

- (i) ϕ set
- (ii) $\{2\}$, therefore, it is not a null set.
- (iii) $\{\phi\}$
- (iv) $\{\phi\}$

2. Which of the following sets are finite or infinite

- (i) Finite
- (ii) Infinite
- (iii) Finite
- (iv) Infinite
- (v) Finite

3. State whether each of the following set is finite or infinite.

- (i) Infinite
- (ii) Finite
- (iii) Infinite
- (iv) Finite
- (v) Infinite

4. In the following, state whether $A=B$ or not:

- (i) $A = \{a, b, c, d\}$
 $B = \{d, c, b, a\}$ $A = B$

$$(ii) \quad A = \{4, 8, 12, 16\} \quad A \neq B \\ B = \{8, 4, 16, 18\}$$

$$(iii) \quad A = \{2, 4, 6, 8, 10\} \quad A = B \\ B = \{x : x \text{ is positive even integer and } x \leq 10\}$$

$$(iv) \quad A = \{x : x \text{ is a multiple of } 10\} \quad A \neq B \\ B = \{10, 15, 20, 25, 30, \dots\}$$

5. Are the following pair of sets equal? Give reasons.

$$(i) \quad A = \{2, 3\} \\ B = \{x : x \text{ is a solution of } x^2 + 5x + 6 = 0\}$$

$$\text{w.k.t } A = \{2, 3\}$$

$$B = \text{w.k.t } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ \text{w.k.t } x = \frac{-5 \pm \sqrt{25 - 4(6)}}{2} \\ = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm \sqrt{1}}{2}$$

$$= \frac{-5 + 1}{2} = \frac{-4}{2} \quad \left| \quad \frac{-5 - 1}{2} = \frac{-6}{2}$$

$$= -2 \quad = -3$$

$$\therefore B = \{-2, -3\}$$

$$\therefore A \neq B$$

(ii) $A = \{x : x \text{ is a letter in the word FOLLOW}\}$
 $B = \{y : y \text{ is a letter in the word WOLF}\}$

$$A = \{F, O, L, W\} \quad B = \{W, O, L, F\}$$

$$\therefore A = B$$

6. From the sets given below, select equal sets:

$$A = \{2, 4, 8, 12\}$$

$$B = \{1, 2, 3, 4\}$$

$$C = \{4, 8, 12, 14\}$$

$$D = \{3, 1, 4, 2\}$$

$$E = \{-1, 1\}$$

$$F = \{0, a\}$$

$$G = \{1, -1\}$$

$$H = \{0, 1\}$$

$$B = D,$$

$$E = G$$

Exercise 1.3

6. Write the following intervals in set-builder form:

(i) $(-3, 0)$ $\{x : x \in \mathbb{R}, -3 < x < 0\}$

(ii) $[6, 12]$ $\{x : x \in \mathbb{R}, 6 \leq x \leq 12\}$

(iii) $(0, 12] \in \mathbb{R}$ $\{x : x \in \mathbb{R}, 0 < x \leq 12\}$

(iv) $[-23, 5)$ $\{x : x \in \mathbb{R}, -23 \leq x < 5\}$

CONTAINED & BELONGS TO

E.g.

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$B = \{6, 7, 8\}$$

$B \subset A$ contained
 element $5 \in A$ belongs to

- * C : contained SET
- * E : belongs to ELEMENT

E.g. $A = \{1, 2, 3, \{4, 5\}\}$

→ elements : 1, 2, 3, {4, 5}

$$\{4, 5\} \in A \text{ (element)}$$

$$\{\{4, 5\}\} \subset A \text{ (set)}$$

E.g. $B = \{\{1, 2\}, \{3, 4\}, 5, \{6, 7\}\}$

$$\{\{1, 2\}\} \subset B$$

$$\{3, 4\} \in B$$

$$\{5\} \subset B$$

$$\{\{6, 7\}\} \subset B$$

Exercise 1.3

1. Make correct statements by filling in the symbols \subset or \notin in the blank spaces:

- (i) $\{2, 3, 4\} \subset \{1, 2, 3, 4, 5\}$
- (ii) $\{a, b, c\} \notin \{b, c, d\}$
- (iii) $\{x : x \text{ is a student of class XI of your school}\} \subset \{x : x \text{ student of your school}\}$
- (iv) $\{x : x \text{ is a circle in the plane}\} \notin \{x : x \text{ is a circle in the same plane, w/ radius 1 unit}\}$

(v) $\{x: x \text{ is a triangle in a plane}\} \neq \{x: x \text{ is a rectangle in the plane}\}$

(vi) $\{x: x \text{ is an equilateral } \Delta \text{ in a plane}\} \subset \{x: x \text{ is a triangle in the same plane}\}$

(vii) $\{x: x \text{ is an even natural no.}\} \subset \{x: x \text{ is an integer}\}$

2. Examine whether the following statements are true or false:

(i) $\{a, b\} \neq \{b, c, a\}$ False

(ii) $\{a, e\} \subset \{x: x \text{ is a vowel in the English alphabet}\}$

(iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$ False

(iv) $\{a\} \subset \{a, b, c\}$ True

(v) $\{a\} \in \{a, b, c\}$ False

(vi) $\{x: x \text{ is an even natural no. less than } 6\} \subset \{x: x \text{ is a natural no. which divides } 36\}$ True

3. Let $A = \{1, 2, \{3, 4\}, 5\}$

(i) $\{3, 4\} \subset A$ False

(ii) $\{3, 4\} \in A$ True

(iii) $\{\{3, 4\}\} \subset A$ True

≠
same?
the

- (iv) $1 \in A$ True
- (v) $1 \subset A$ False
- (vi) $\{1, 2, 5\} \subset A$ True
- (vii) $\{1, 2, 5\} \in A$ False
- (viii) $\{1, 2, 3\} \subset A$ False
- (ix) $\emptyset \in A$ False
- (x) $\emptyset \subset A$ True
- (xi) $\{\emptyset\} \subset A$ False

4. Write down all the subsets of the following sets
 $2^n(P(A))$

(i) $\{a\}$ $\{\{a\}, \emptyset\}$

(ii) $\{a, b\}$ $\{\{a, b\}, \emptyset, \{a\}, \{b\}\}$

(iii) $\{1, 2, 3\}$ $\{\{1, 2, 3\}, \emptyset, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1\}, \{2\}, \{3\}\}$

(iv) \emptyset $\{\emptyset\}$

Homework: Exercise 1.3 \rightarrow 5, 7, 8

5. Write the following as intervals:

(i) $\{x : x \in \mathbb{R}, -4 < x \leq 6\}$ $(-4, 6]$

(ii) $\{x : x \in \mathbb{R}, -12 < x < -10\}$ $(-12, -10)$

(iii) $\{x : x \in \mathbb{R}, 0 \leq x < 7\}$ $[0, 7)$

(iv) $\{x : x \in \mathbb{R}, 3 \leq x \leq 4\}$ $[3, 4]$

7. What universal set(s) would you propose for each of the following:

(i) The set of right triangles.
The set of triangles

(ii) The set of isosceles triangles.
The set of triangles.

8. Given the sets $A = \{1, 3, 5\}$,

$$B = \{2, 4, 6\},$$

$$C = \{0, 2, 4, 6, 8\},$$

which of the following maybe considered as universal set (S) for all the three sets A, B and C

(iii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

30.05.2023

VENN DIAGRAM

UNION 'U' merge

E.g.

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{4, 5, 6, 7, 8\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

Definition:

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

INTERSECTION '∩' only common

E.g. $A = \{a, b, c, d, e\}$

$$B = \{e, f, g, h, i\}$$

$$A \cap B = \{e\}$$

SOME PROPERTIES OF UNION :

- (i) $A \cup B = B \cup A$ [commutative law]
- (ii) $(A \cup B) \cup C = A \cup (B \cup C)$ [associative law]
- (iii) $A \cup \phi = A$ [law of identity element,
 ϕ is the identity of \cup]
- (iv) $A \cup A = A$ [Idempotent law]
- (v) $\cup \cup A = \cup$ [Law of \cup]

SOME PROPERTIES OF INTERSECTION :

- (i) $A \cap B = B \cap A$ [commutative law]
- (ii) $(A \cap B) \cap C = A \cap (B \cap C)$ [associative law]
- (iii) $\phi \cap A = \phi, \cup \cap A = A$ [Law of ϕ and \cup]
- (iv) $A \cap A = A$ [Idempotent law]
- (v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
[Distributive law]
- i.e., \cap distributes over \cup

Exercise 1.4

1. Find the union:

$$(i) \quad X = \{1, 3, 5\}$$

$$Y = \{1, 2, 3\}$$

$$X \cup Y = \{1, 2, 3, 5\}$$

$$(ii) \quad A = \{a, e, i, o, u\}$$

$$B = \{a, b, c\}$$

$$A \cup B = \{a, e, i, o, u, b, c\}$$

$$(iii) \quad A = \{x : x \text{ is a natural no. \& multiple of } 3\}$$

$$B = \{x : x \text{ is a natural no. less than } 6\}$$

$$A = \{3, 6, 9, 12, 15, \dots\}$$

$$B = \{1, 2, 3, 4, 5\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 9, 12, \dots\}$$

$$(iv) \quad A = \{x : x \text{ is a natural no. \& multiple of } 3\}$$

$$B = \{x : x \text{ is a natural no. \& } 1 < x \leq 6\}$$

$$B = \{x : x \text{ is a natural no. \& } 6 < x < 10\}$$

$$A = \{2, 3, 4, 5, 6\}$$

$$B = \{7, 8, 9\}$$

$$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$$

$$(v) \quad A = \{1, 2, 3\}$$

$$B = \{\emptyset\}$$

$$A \cup B = \{1, 2, 3\}$$

2. Let $A = \{a, b\}$, $B = \{a, b, c\}$.
 Is $A \subset B$?
 What is $A \cup B$?

Yes $A \subset B$

$$A \cup B = \{a, b, c\} = B$$

- * If $A \subset B$, then
 $A \cup B = B$.
 Similarly, if $B \subset A$, then
 $A \cup B = A$.

3. If A & B are two sets such that $A \subset B$,
 then what is $A \cup B$?

W.K.T $A \subset B$,

$$\boxed{\therefore A \cup B = B}$$

4. If $A = \{1, 2, 3, 4\}$,
 $B = \{3, 4, 5, 6\}$
 $C = \{5, 6, 7, 8\}$,
 $D = \{7, 8, 9, 10\}$; find

- (i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$
 (ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
 (iii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$
 (iv) $B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$
 (v) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
 (vi) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
 (vii) $B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

$$6. \text{ If } A = \{3, 5, 7, 9, 11\},$$

$$B = \{7, 9, 11, 13\},$$

$$C = \{11, 13, 15\},$$

$$D = \{15, 17\}; \text{ find}$$

$$(i) A \cap B = \{7, 9, 11\}$$

$$(ii) B \cap C = \{11, 13\}$$

$$(iii) A \cap C \cap D = \emptyset$$

$$(iv) A \cap C = \{11\}$$

$$(v) B \cap D = \emptyset$$

$$(vi) A \cap (B \cup C) = \{7, 9, 11\}$$

$$(vii) A \cap D = \emptyset$$

$$(viii) A \cap (B \cup D) =$$

$$(B \cup D) = \{7, 9, 11, 13, 15, 17\}$$

$$A \cap (B \cup D) = \{7, 9, 11\}$$

$$(ix) (A \cap B) \cap (B \cup C)$$

$$(A \cap B) = \{7, 9, 11\}$$

$$(B \cup C) = \{7, 9, 11, 13, 15\}$$

$$(A \cap B) \cap (B \cup C) = \{7, 9, 11\}$$

$$(x) (A \cup D) \cap (B \cup C)$$

$$(A \cup D) = \{3, 5, 7, 9, 11, 15, 17\}$$

$$(B \cup C) = \{7, 9, 11, 13, 15\}$$

$$(A \cup D) \cap (B \cup C) = \{7, 9, 11, 15\}$$

7. If

$$A = \{x : x \text{ is a natural no.}\}$$

$$B = \{x : x \text{ is an even natural no.}\}$$

$$C = \{x : x \text{ is an odd natural no.}\}$$

$$D = \{x : x \text{ is a prime no.}\}$$

find,

$$(i) \quad A = \{1, 2, 3, 4, 5, 6, \dots\}$$

$$B = \{2, 4, 6, 8, \dots\}$$

$$C = \{1, 3, 5, 7, 9, \dots\}$$

$$D = \{2, 3, 5, 7, 11, \dots\}$$

$$(i) \quad A \cap B = \{2, 4, 6, 8, 10, 12, \dots\}$$

$$(ii) \quad A \cap C = \{1, 3, 5, 7, 9, \dots\}$$

$$(iii) \quad A \cap D = \{2, 3, 5, 7, 11, 13, \dots\}$$

$$(iv) \quad B \cap C = \phi$$

$$(v) \quad B \cap D = \{2\}$$

$$(vi) \quad C \cap D = \{3, 5, 7, 11, 13, 17, \dots\}$$

8. Which of the following pairs of sets are disjoint

$$(i) \quad \{1, 2, 3, 4\}$$

$$\{4, 5, 6\}$$

$$\text{Common element} = \{4\}$$

\therefore It is not ~~not~~ disjoint

$$(ii) \quad \{a, e, i, o, u\}$$

$$\{c, d, e, f\}$$

$$\text{Common element} = \{e\} \therefore \text{It is not disjoint}$$

$$(iii) \quad \text{Yes, it is disjoint.}$$

- vi) $D - A = \{5, 10, 20\}$
 vii) $B - C = \{4, 8, 16, 20\}$
 viii) $B - D = \{5, 10, 15\}$
 ix) $C - B = \{2, 6, 10, 14\}$
 x) $D - B = \{5, 10, 15\}$
 xi) $C - D = \{2, 4, 6, 8, 12, 14, 16\}$
 xii) $D - C = \{5, 15, 20\}$

11. Q. If R is the set of real no.s,
 Q is the set of rational no.s,
 then what is $R - Q$?

$$R - Q = \{x : x \text{ is an irrational no.}\}$$

COMPLEMENT OF A SET

→ Let U be the universal set & A a subset of U . Then the complement of A is the set of all elements of U which are not elements of A .

→ Symbolically, we write A' to denote the complement of A with respect to U .

Thus,

$$A' = \{x : x \in U \text{ and } x \notin A\}$$

Obviously $A' = U - A$

$$(A')' = A$$

06/06/23

SOME PROPERTIES OF COMPLEMENT SETS

1. Complement laws:

$$i) A \cup A' = U$$

$$ii) A \cap A' = \phi \text{ (disjoint)}$$

2. De Morgan's laws:

$$i) (A \cup B)' = A' \cap B'$$

$$ii) (A \cap B)' = A' \cup B'$$

3. Law of double complementation:

$$(A')' = A$$

4. Laws of empty set & universal set

$$\phi' = U$$

$$U' = \phi$$

Exercise 1.5

1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$A = \{1, 2, 3, 4\}$$

$$B = \{2, 4, 6, 8\}$$

$$C = \{3, 4, 5, 6\}$$

Ques

$$i) A' = U - A$$

$$A' = \{5, 6, 7, 8, 9\}$$

$$ii) B' = U - B$$

$$B' = \{1, 3, 5, 7, 9\}$$

$$\text{iii) } (A \cup C)'$$

$$A \cup C = \{1, 2, 3, 4, 5, 6\}$$

$$(A \cup C)' = U - (A \cup C)$$

$$(A \cup C)' = \{7, 8, 9\}$$

$$\text{iv) } (A \cup B)'$$

$$A \cup B = \{1, 2, 3, 4, 6, 8\}$$

$$(A \cup B)' = U - (A \cup B)$$

$$(A \cup B)' = \{5, 7, 9\}$$

$$\text{v) } (A')' = A \text{ (w.k.t.)}$$

$$(A')' = \{1, 2, 3, 4\}$$

$$\text{vi) } (B - C)'$$

$$B - C = \{2, 8\}$$

$$(B - C)' = U - (B - C)$$

$$(B - C)' = \{1, 3, 4, 5, 6, 7, 9\}$$

2. If $U = \{a, b, c, d, e, f, g, h\}$,
find the complements of the following sets:

$$\text{i) } A = \{a, b, c\}$$

$$A' = \{d, e, f, g, h\}$$

$$\text{ii) } B = \{d, e, f, g\}$$

$$B' = \{a, b, c, h\}$$

$$\text{iii) } C = \{a, c, e, g\}$$

$$C' = \{b, d, f, h\}$$

$$\text{iii) } D = \{f, g, h, a\}$$

$$D' = \{e, d, c, b\}$$

$$4. \text{ If } U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

$$A = \{2, 4, 6, 8\}$$

$$B = \{2, 3, 5, 7\}$$

Verify that,

$$i) (A \cup B)' = A' \cap B'$$

$$ii) (A \cap B)' = A' \cup B'$$

$$i) \text{ LHS} = (A \cup B)' =$$

$$A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$$

$$(A \cup B)' = \{1, 9\} = \text{LHS}$$

$$\text{RHS} = A' \cap B'$$

$$A' = \{1, 3, 5, 7, 9\}$$

$$B' = \{1, 4, 6, 8, 9\}$$

$$A' \cap B' = \{1, 9\} = \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence, Proved.

$$ii) \text{ LHS} = (A \cap B)' =$$

$$A \cap B = \{2\}$$

$$(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\} = \text{LHS}$$

$$\text{RHS} = A' \cup B' =$$

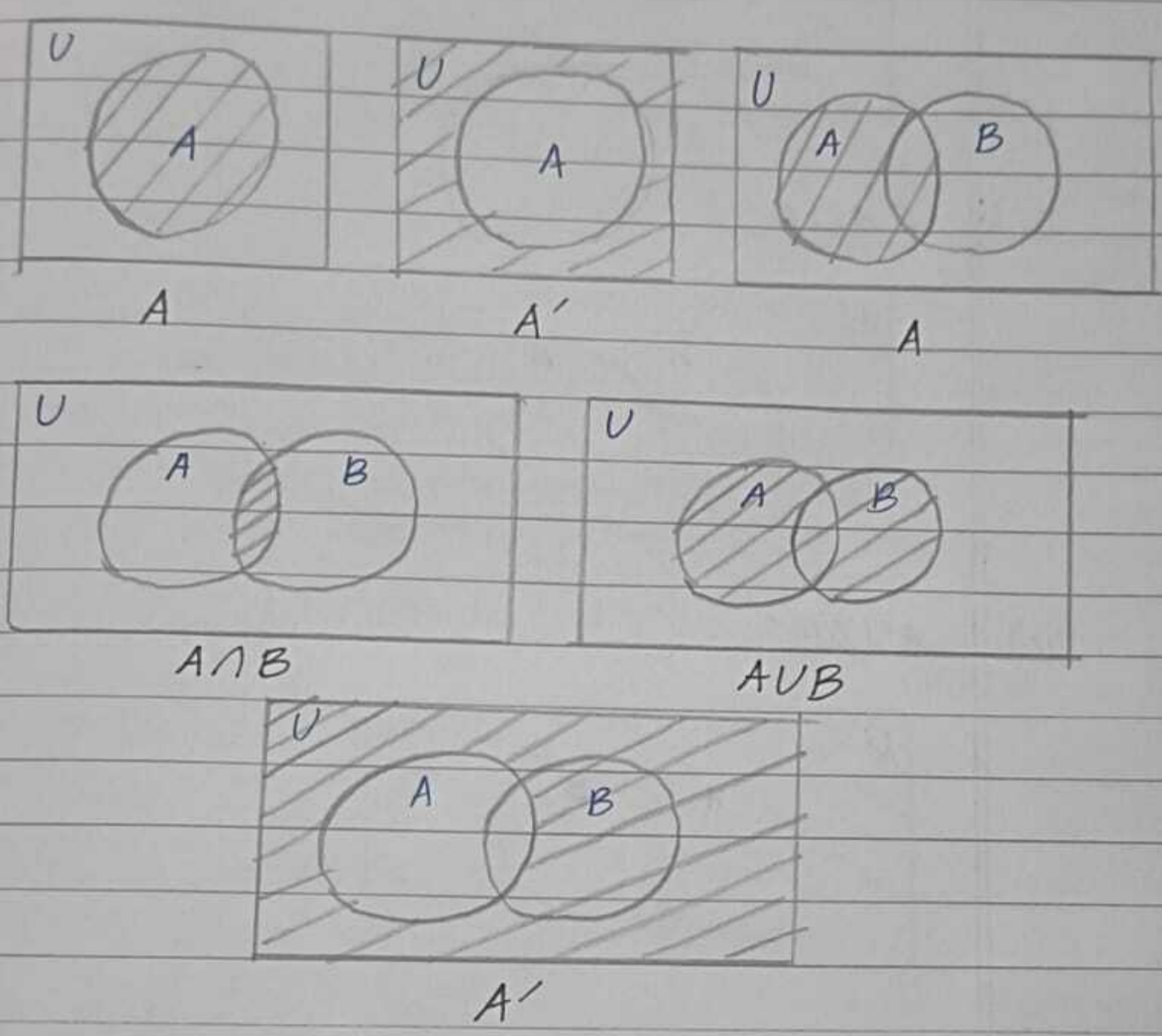
$$A' = \{1, 3, 5, 7, 9\}$$

$$B' = \{1, 4, 6, 8, 9\}$$

$$A' \cup B' = \{1, 3, 4, 5, 6, 7, 8, 9\} = \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS} \quad \text{Hence, proved}$$

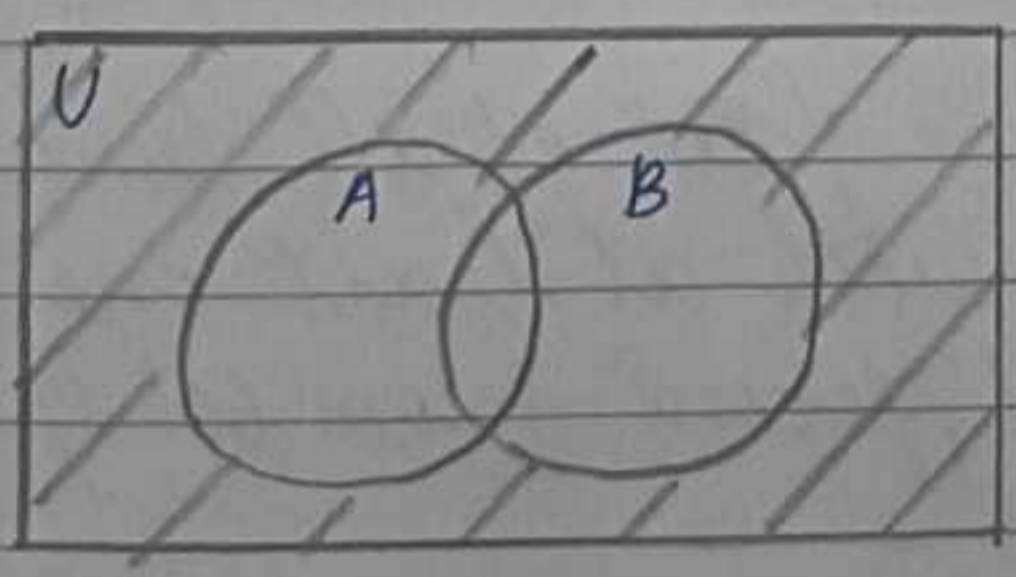
Venn Diagrams



Exercise 1.5

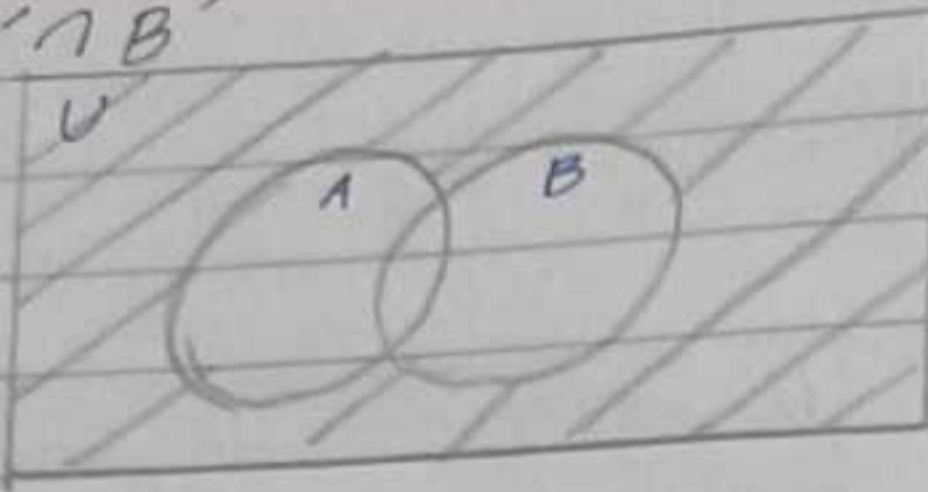
5. Draw appropriate Venn diagram for each of the following:

i) $(A \cup B)'$

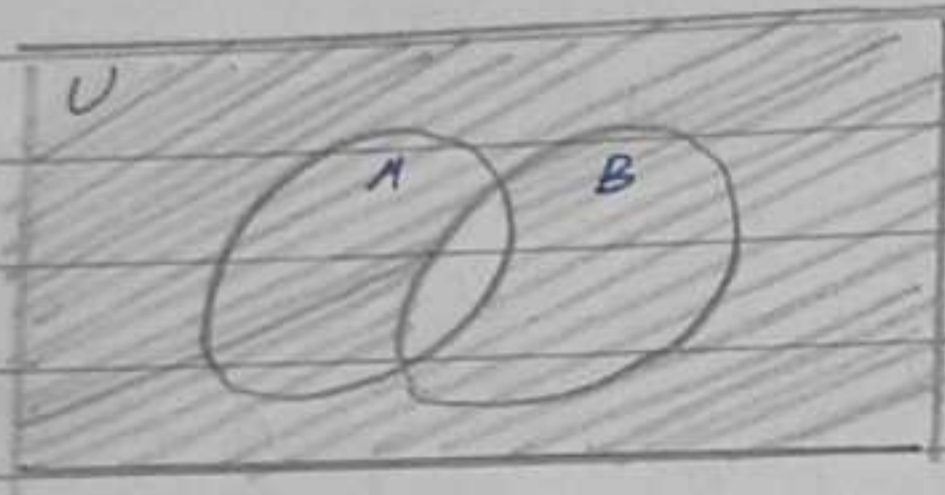


ii) $A' \cap B'$

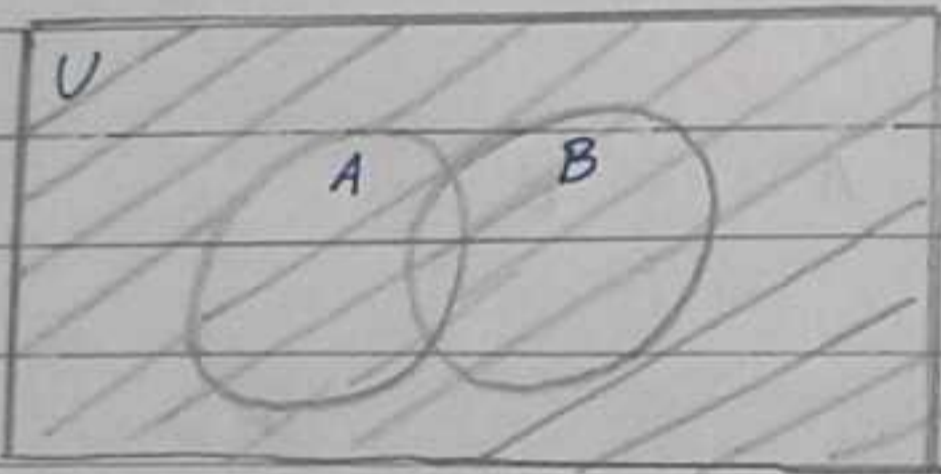
(ii)



iii) $(A \cap B)'$



iv) $A' \cup B'$



Homework: Ex. 1.5 — Q6.

6. Let U be the set of all triangles in a plane. If A is a set of all triangles with at least one angle different from 60° , what is A' ?

Given:

$A = \{ \text{set of all triangles with at least one angle different from } 60^\circ \}$

$\therefore A' = \{ \text{set of equilateral triangles} \}$

\therefore In equilateral Δ s all angles are 60° .